13 **Calcula los siguientes límites:**

a) $\lim\limits_{x \to 0} \dfrac{4x}{x^2 - 2x}$ b) $\lim\limits_{x \to 0} \dfrac{2x^2 + 3x}{x}$ c) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$ d) $\lim\limits_{x \to -1} \dfrac{x^3 + 1}{x^2 + x}$

a) $\lim\limits_{x \to 0} \dfrac{4x}{x^2 - 2x} = \lim\limits_{x \to 0} \dfrac{4x}{x(x-2)} = -2$

b) $\lim\limits_{x \to 0} \dfrac{2x^2 + 3x}{x} = \lim\limits_{x \to 0} \dfrac{x(2x+3)}{x} = 3$

c) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1} = \lim\limits_{x \to 1} \dfrac{(x+1)(x-1)}{(x-1)} = 2$

d) $\lim\limits_{x \to -1} \dfrac{x^3 + 1}{x^2 + x} = \lim\limits_{x \to -1} \dfrac{x^3 + 1}{x^2 + x} = \lim\limits_{x \to -1} \dfrac{(x+1)(x^2 - x + 1)}{x(x+1)} = \dfrac{3}{-1} = -3$

14 **Resuelve los siguientes límites y representa los resultados que obtengas:**

a) $\lim\limits_{x \to 1} \dfrac{x^2}{x - 1}$

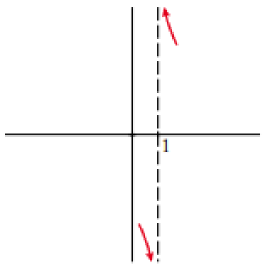
b) $\lim\limits_{x \to 0} \dfrac{x^2 + x}{x^2}$

c) $\lim\limits_{x \to -2} \dfrac{x^2}{x^2 + 2x}$

d) $\lim\limits_{x \to 0} \dfrac{x^3}{x^4 - 10x^2}$

a) $\lim\limits_{x \to 1} \dfrac{x^2}{x - 1} = \dfrac{1}{0} = \pm\infty$

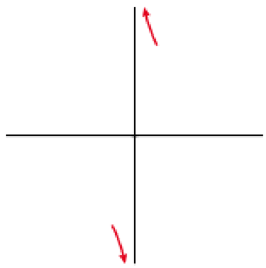
- Si $x \to 1^- \to \left(f(x) = \dfrac{+}{-} = -\right)\ f(x) \to -\infty$
- Si $x \to 1^+ \to \left(f(x) = \dfrac{+}{+} = +\right)\ f(x) \to +\infty$



b) $\lim\limits_{x \to 0} \dfrac{x^2 + x}{x^2} = \dfrac{0}{0} \to$ Indeterminación.

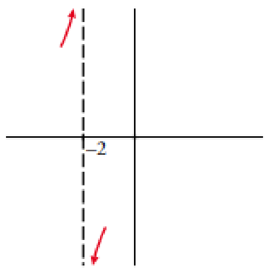
$\dfrac{x^2 + x}{x^2} = \dfrac{x(x+1)}{x^2} = \dfrac{x+1}{x}$; $\lim\limits_{x \to 0} \dfrac{x^2 + x}{x^2} = \lim\limits_{x \to 0} \dfrac{x+1}{x} = \dfrac{1}{0} = \infty$

- Si $x \to 0^- \to \left(f(x) = \dfrac{+}{-} = -\right)\ f(x) \to -\infty$
- Si $x \to 0^+ \to \left(f(x) = \dfrac{+}{+} = +\right)\ f(x) \to +\infty$



c) $\lim\limits_{x \to -2} \dfrac{x^2}{x^2 + 2x} = \dfrac{4}{0} = \pm\infty$

- Si $x \to -2^- \to \left(f(x) = \dfrac{+}{+} = +\right)\ f(x) \to +\infty$
- Si $x \to -2^+ \to \left(f(x) = \dfrac{+}{-} = -\right)\ f(x) \to -\infty$



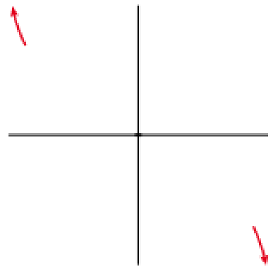
d) $\lim\limits_{x \to 0} \dfrac{x^3}{x^4 - 10x^2} = \dfrac{0}{0} \to$ Indeterminación.

$$\frac{x^3}{x^4 - 10x^2} = \frac{x^3}{x^2(x^2 - 10)} = \frac{x}{x^2 - 10}$$

$$\lim_{x \to 0} \frac{x^3}{x^4 - 10x^2} = \lim_{x \to 0} \frac{x}{x^2 - 10} = 0$$

$f(0)$ no está definido.

Si $x < 0$, $f(x) > 0$ y si $x > 0$, $f(x) < 0$



15 Calcula los siguientes límites y representa los resultados que obtengas:

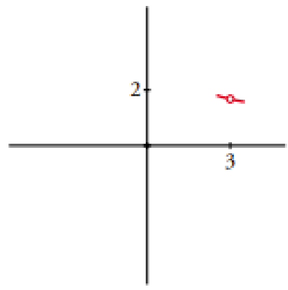
a) $\lim\limits_{x \to 3} \dfrac{x^2 - x - 6}{x^2 - 3x}$ **b)** $\lim\limits_{x \to 1} \dfrac{x^2 - 3x + 2}{x^2 - 2x + 1}$ **c)** $\lim\limits_{x \to 0} \dfrac{x^2 - 2x}{x^3 + x^2}$ **d)** $\lim\limits_{x \to -1} \dfrac{x^3 + x^2}{x^2 + 2x + 1}$

a) $\lim\limits_{x \to 3} \dfrac{x^2 - x - 6}{x^2 - 3x} = \dfrac{0}{0} \to$ Indeterminación.

$$\frac{x^2 - x - 6}{x^2 - 3x} = \frac{(x+2)(x-3)}{x(x-3)} = \frac{x+2}{x}$$

$$\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 3x} = \lim_{x \to 3} \frac{x+2}{x} = \frac{5}{3}$$

Dando a x valores próximos a 3 podemos averiguar cómo se acerca por ambos lados.



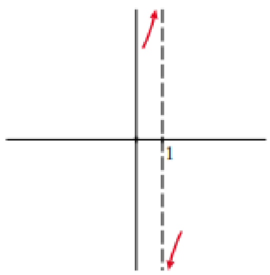
b) $\lim\limits_{x \to 1} \dfrac{x^2 - 3x + 2}{x^2 - 2x + 1} = \dfrac{0}{0} \to$ Indeterminación.

Simplificamos: $\dfrac{x^2 - 3x + 2}{x^2 - 2x + 1} = \dfrac{(x-2)(x-1)}{(x-1)^2} = \dfrac{x-2}{x-1}$

$$\lim_{x \to 1} \frac{x^2 - 3x + 2}{x^2 - 2x + 1} = \frac{0}{0} = \lim_{x \to 1} \frac{x-2}{x-1} = \infty$$

• Si $x \to 1^- \to \left(f(x) = \dfrac{-}{-} = +\right)$ $f(x) \to +\infty$

• Si $x \to 1^+ \to \left(f(x) = \dfrac{-}{+} = -\right)$ $f(x) \to -\infty$



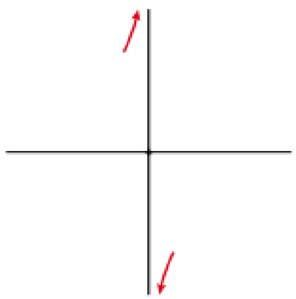
c) $\lim\limits_{x \to 0} \dfrac{x^2 - 2x}{x^3 + x^2} = \dfrac{0}{0} \to$ Indeterminación.

$$\frac{x^2 - 2x}{x^3 + x^2} = \frac{x(x-2)}{x^2(x+1)} = \frac{x-2}{x(x+1)}$$

$$\lim_{x \to 0} \frac{x^2 - 2x}{x^3 + x^2} = \lim_{x \to 0} \frac{x-2}{x(x+1)} = \frac{-2}{0} = \pm\infty$$

• Si $x \to 0^- \to \left(f(x) = \dfrac{-}{-} = +\right)$ $f(x) \to +\infty$

• Si $x \to 0^+ \to \left(f(x) = \dfrac{-}{+} = -\right)$ $f(x) \to -\infty$



d) $\lim_{x \to -1} \frac{x^3 + x^2}{x^2 + 2x + 1} = \frac{0}{0} \rightarrow$ Indeterminación.

$$\frac{x^3 + x^2}{x^2 + 2x + 1} = \frac{x^2(x+1)}{(x+1)^2} = \frac{x^2}{x+1}$$

$$\lim_{x \to -1} \frac{x^3 + x^2}{x^2 + 2x + 1} = \lim_{x \to -1} \frac{x^2}{x+1} = \frac{1}{0} = \pm\infty$$

- Si $x \to -1^- \rightarrow \left(f(x) = \frac{+}{-} = - \right) f(x) \to -\infty$
- Si $x \to -1^+ \rightarrow \left(f(x) = \frac{+}{+} = + \right) f(x) \to +\infty$